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Fuzzy Logics and Fuzzy Set Theory based Multi-objective Decision Making, Modeling Tool generalized for Planning for Sustainable Water Supply

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Abstract

The multi-level fuzzy logics model developed by fuzzy logics is based on the weightages and rankings with establishment of benchmark and finally gives the results for sustainable water supply by fuzzy decision making.The model developed by this author is a one-point scale. The possible alternative, factors and preferences are from 0, 0.1,0.2,0.3,0.4…1, so that the instant decisions should be made based on preferences of experts or respondents. The possible alternatives, factors and preferences are rated from 0,0.1,0.2,0.3,0.4…1 so that appropriate decisions should be finalized to carry out field monitoring for sustainable water supply systems.

1. INTRODUCTION

Many simple decision processes are based on a single factor, such as minimizing cost, maximizing profit, minimizing run time, energy efficiency, pollution control, etc; often however, decisions must be made in an environment where more than one objective function governs constraints on the problem, and the relative value of each of these factors are different. For example,when planning a new colony, we want simultaneously to minimize costs, maximize convenience, maximize city infrastructure (C_i), and maximizing reliability. Moreover, suppose the cost is the most important of all factors and the other three carry lesser but equal weight when compared with cost. Two primary issues in multi-objective decision-making are to be acquired meaningfully for getting information regarding the satisfaction by the various choices (alternatives) and to rank or weight the relative importance of each of these factors. The approach illustrated in this section defines decision calculus that requires only ordinal information on the ranking of preferences and importance weights (Yager, 1981).

The typical multi factor decision problem involves the selection of one alternative, a_i, from a universe of alternatives A given a collection, or set, say (O), of criteria or factors that are important to the decision maker. We want to evaluate how well each alternative, or choice satisfies each factor, and we wish to combine the weighted factors into an overall decision function in some possible way. This decision function essentially represents a mapping of the alternatives in A to an ordinal set of ranks. This process naturally requires subjective information from the decision authority concerning the importance of each factor. Ordinal orderings of these hierarchies are usually the easiest to obtain. Numerical values,

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ratios and intervals expressing the importance of each factor are difficult to extract and if attempted and then subsequently altered can often lead to results inconsistent with the intuition of the decision maker.

To develop this calculus we require some definitions. Define a universe of n alternatives, $A = \{a_1, a_2, a_3------a_n\}$ and are set of r factors, $F = \{f_1, f_2, f_3...f_r\}$. Let f_i indicates the I_{th} factor then the degree of membership of alternatives a in f_i denotes μf_i (a) is the degree to which alternative a satisfies the criteria specified for this factor. We seek a decision function that simultaneously satisfies all of the decision factors. Hence the decision function D is given by intersection of the entire factor sets,

$D = F_1 \cap F_2 \cap F_3 ....F_r$ ---------------(I)

Therefore the grade of memberships that the decision functions D has for each alternative a is given by

$\mu D (a) = \min [\mu f_1 (a_1), \mu f_2 (a_2), \mu f_3 (a_3)............ \mu f_r(a_n)]$ -------------(II)

The optimum decision, a*, will then be the alternative that satisfies

$\mu D (a^*) = \max. (\mu D (a))$ ---------------(III)

$a \in A$

We now define a set of preferences, {P}, which we will constrain to being linear and ordinal. Elements of this preference set can be linguistic values such as none, low, medium, high, absolute or perfect; or they could be values on the interval [0,1]; or they could be values on any other linearly ordered scale, e.g. [-1, 1], [1,10], etc. These preferences will be attached to each of the factors to quantify the decision makers feeling about the influence that each objective should have on the chosen alternative. Let the parameter b_1 be contained on the set of preferences {p} where I = 1,2,3...r. Hence we have for each factor a measure of how important it is to the decision maker for a given decision.

The decision function, D, now takes a more general form when each objective is associated with a weight expressing its importance to the decision maker. This function is represented as the intersection of r attributes, denoted as a decision measure, $M (f_1, b_1)$, involving factors and preferences.

$D = M (f_1, b_1) \cap M (f_2, b_2) \cap M (f_3, b_3) \cap ... \cap M (f_r, b_r)$ -------------- (IV)

A key question is what operation should relate to each factor, f_i, and its importance, b_i that preserves the linear ordering required of the preference set, and at the same time relates the two quantities in a logical way where negotiation is also accommodated. It turns out that the classical implication operator satisfies all of these requirements. Hence the decision measure for a particular alternative, a, can be replaced with a classical implication of the form

$M (F_i (a), b_i) = bi \rightarrow F_i (a) = -b_i \cup F_i (a)$ --------------------(V)

Justification of the implication as an appropriate measure can be developed using an intuitive argument (Yager, 1981). The statement 'b_i implies f_i' indicates a unique relationship between a preference and its associated factor. Whereas various factors can have the same preference weighting in a coordinal sense, they will be unique in an ordinal sense even though the equality situation $b_i = b_j$ for $i \neq j$ can exist for some factors. Ordering will be preserved because $b_i \geq b_j$ will contain the equality case as subset.

Therefore a reasonable decision model will be the joint intersection of r decision measures,

$D = r\cap i=1 \{-b_i \cup F_i\}$ --------------------(VI)

And the optimum solution, a* is the alternative that maximizes D. if we design

$C_i = -b_i \cup F_i$ hence $\mu_{ci} (a) = \max [\mu_{bi} - (a), \mu_{oi} (a)]$ --------------------(VII)

Then the optimum solution, expressed in membership form is given by

$\mu_D (a^*) = \max a \in A [\min \{\mu c_1 (a), \mu c_2 (a),..\mu c_r(a)\}]$ -------------------(VIII)

This model is instinctive in the following manner. As the i^{th} objective becomes more important in the final decision, b_i increases, causing $-b_i$ to decrease, which in turn causes $C_i(a)$ to decrease, thereby increasing the likelihood that $C_i(a) = F_i(a)$, where now $F_i(a)$ will be the value of the decision function, D, representing the alternative a see eq.(VI). As we repeat this process for other alternatives, an eq. VIII reveals that the largest value $f_1(a)$ for other alternatives will eventually result in the choice of the optimum solution, a*. This is exactly how we would want the process to work.

For particular factor, the negation of its importance (preference) acts as a barrier such that all ratings of alternatives below that barrier become equal to the value of that barrier (Yager, 1981). Here we disregard all distinctions less than the barrier while keeping distinctions above this barrier. This process is similar to the grading practice of the academics that lump all students whose class average falls below 60 percent into the f category while keeping distinctions of ABC and D for students above this percentile. However, in the decision module measured here this barrier varies, depending upon the preferences, (importance) of the objective to the decision maker. The more important is the factor, the lower is the barrier and thus the more levels of distinctions there are. As a factor becomes less important the distinction barrier increases, which lessens the penalty to the factor. In the limit, if the factor becomes totally unimportant, then the barrier is raised to its highest level and all alternatives are given the same weight and no distinction is made. Conversely if the factor becomes the most important, all distinctions remain. In sum, the more important a factor is in the decision process, the more significant is its effect on the decision function, D.

A special procedure should be followed in the event of numerical tie between two or more alternatives (Yager, 1981). If two alternatives x and y are tied, their respective decision values are equal i.e. D (x) = D (y) = max aЄA [D (a)], where a = x = y. Since D (a) = min_i [C_i(a)] there exists some alternative k such that Ck(x) = D(x) and some alternative g such that Cg (y) = D (y). Let

Ď (x) = $min_i \neq k$ [C_i (x)] and Ď (y) = $min_i \neq g$ [C_i (y)] ------------------- (IX)

Then, we compare Ď (x) and Ď (y) and if, for e.g. Ď (x) > Ď (y) we select x as our optimum alternative. However if a tie still persists i.e. if Ď (x) = Ď (y) then there exists some other alternatives j and h such that Ď (x) = Cj(x) = Ď(y) = Ch(y).

Then we formulate

ĎĐ (x) = $min_i \neq k$ [C_i (x)] and ĎĐ(y) = $min_i \neq g$, h [C_i(y)] ---------------(X)

And compare ĎĐ (x) and ĎĐ (y). The tie breaking procedure continues in this manner until an unambiguous optimum alternative emerges or all of the alternatives are exhausted. In the latter case where a tie still results some other tie breaking procedure such as refinement in the preference scales can be used.

2. WATER SUPPLY SYSTEM ANALYSIS

Alternative (A) = a_1, a_2, a_3 -----------

Factors (F) = f_1, f_2, f_3 ----------

A = { U_{lws}, K_{ws}, N_{ws}, G_{ws}, T_{ws}}

= {a_1, a_2, a_3, a_4, a_5}

F = vegetation, lithology, lineament, soil texture, lithology, sandstone basalt, lineaments

= {Veg, S_{lt}, L_{ist}, L_i} = f_1, f_2, f_3, f_4

Prefrences P = {b_1, b_2, b_3, b_4} $\rightarrow$ 0,1

For case study analysis of water supply schemes in Bhopal, I first rated the various water supply schemas with respect to the factors expressed in Zade's rotation

2.1 Physical Factors of Decision Making

- Energy efficiency in terms of cost: C_s
- Pollution control safe limit 10 per 100ml i.e. 10/100ml. P_c
- Flood risk areas/slope/valley flow/low laying/topography: S_l

- Proximity to water supply source distance: P_r
- Population/density: P_d
- Tax collection/ revenue generation: T_{crg}
- Environmental impact/ rainfall: R_f
- Vegetation/ lineaments/ pediments/ fractures/ recharge zones: W_s
- Lithology favorable for ground water recharge: L_i
- Social factors in terms of Housing viz MIG, HIG, LIG, EWS, SLUMS: S_c
- Waste water treatment: G_{wr}
- Standard design: S_d

Alternatives (A) = q_1, q_2, q_3…

A= $\{U_{lws}, K_{ws}, N_{ws}, G_{ws}, T_{ws}\}$

Attributes =Cst, Pc, Sl, Pr, Pd, Tcrg, Rf, Ws, Li, Sc, Gwr, Sd

The conventional water supply schemes followed in Bhopal nowadays is too expensive, so we must prevent huge expenditure expended upon water supply to ward 54, Bhopal city and must save this huge amount for exhibiting other water augmentation schemes and must retain distribution losses of water to maintain sustainable water supply of an area around a given ward (site) (also see Addams (1994). We therefore must decide which type of water supply scheme would be implemented so as to achieve water supply. Among many alternative designs available I have scheduled the list of candidate parameters water supply scheme to give (i) Upper lake water supply (U_{lws}) (ii) Kolar water supply (K_{ws}) (iii) Narmada water supply (N_{ws}) (iv) Ground water (G_{ws}) (v) Tanker water (T_{ws}).

The Bhopal municipal corporation (decision maker) has defined four objects that impact this decision (i) the cost of water supply (cost) (ii) the maintenance of water supply infrastructure (main) (iii) whether the design is a standard one (S_d) (iv) the environmental impact of water supply system, more over the cooperator also decides to rank his preferences for these factors on the unit interval. Hence I have setup this problem as follows:

Alternatives (A) = q_1, q_2, q_3… factors f_1, f_2, f_3…f_r

A = $\{U_{lws}, K_{ws}, N_{ws}, G_{ws}, T_{ws}\}$

= $\{a_1, a_2, a_3, a_4, a_5\}$

F = {cost, main, SD, environmental} =f_1, f_2, f_3, f_4

P = $\{b_1, b_2, b_3, b_4\}$ 0, 1

From case studies of water supply schemes of Bhopal, the planner first rates the various water supply courses with respect to the objective given above. These membership functions for each of two alternatives are shown in the Fig. 1 graphically.

The ratings are Fuzzy sets expressed in Zade's notation

$F_1 = \{0.9/U_{lws} + 0.8/K_{ws} + 0.5/N_{ws} + 0.8/G_{ws} + 0.1/T_{ws}\}$

$F_2 = \{0.5/U_{lws} + 0.7/K_{ws} + 0.9/N_{ws} + 0.4/G_{ws} + 0.4/T_{ws}\}$

$F_3 = \{0.9/U_{lws} + 0.8/K_{ws} + 0.6/N_{ws} + 0.9/G_{ws} + 0.7/T_{ws}\}$

$F_4 = \{0.8/U_{lws} + 0.6/K_{ws} + 0.5/N_{ws} + 0.4/G_{ws} + 0.1/T_{ws}\}$

These preferences for each of the four factors as shown in Fig. 2 above from these preference values, the following calculation results:

$b_1 = 0.9$, $b_2 = 0.8$, $b_3 = 0.5$, $b_4 = 0.8$, $b^{-1} = 0.1$, $b^{-2} = 0.2$, $b^{-3} = 0.5$, $b^{-4} = 0.2$,

$D(a_1) = D(U_{ls}) = (b^{-1} \cup f_1) \cap (b^{-2} \cup f_2) \cap (b^{-3} \cup f_3) \cap (b^{-4} \cup f_4)$

$= (0.1 \cup 0.9) \cap (0.2 \cup 0.5) \cap (0.5 \cup 1) \cap (0.2 \cup 0.8)$

$= 0.9 \cap 0.5 \cap 1 \cap 0.8 = 0.5$

$D(a_2) = D(K_{ws}) = (0.1 \cup 0.8) \cap (0.2 \cup 0.7) \cap (0.5 \cup 0.8) \cap (0.2 \cup 0.6)$

$= 0.8 \cap 0.7 \cap 0.8 \cap 0.6 = 0.6$

Fig. 1: Membership Functions

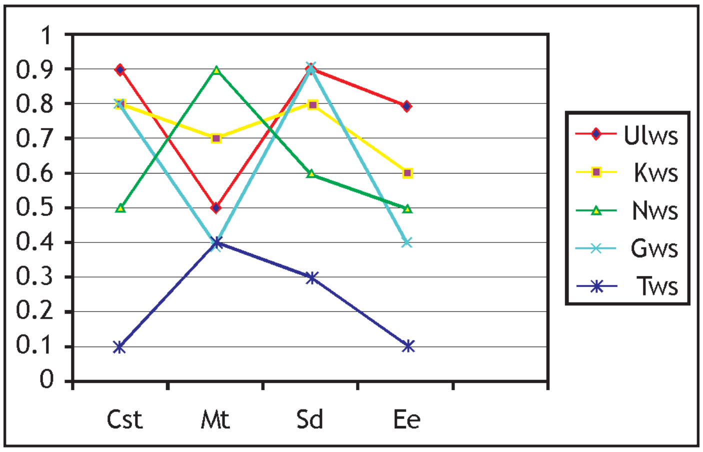
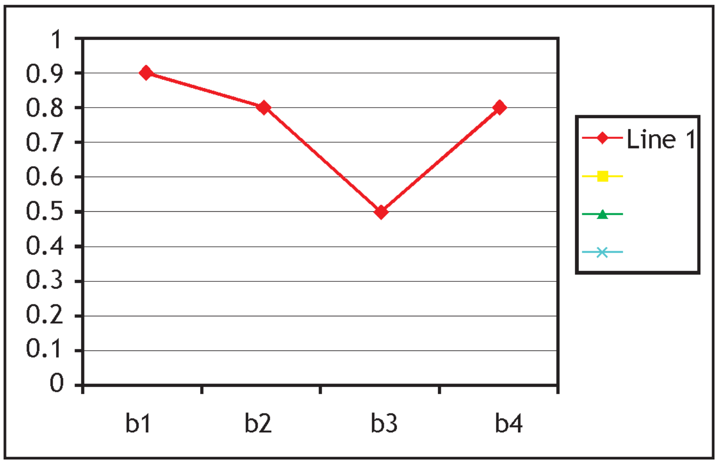


Fig. 2: Preferences Value



$D(a_3) = D(N_{ws}) = (0.1 \cup 0.5) \cap (0.2 \cup 0.9) \cap (0.5 \cup 0.6) \cap (0.2 \cup 0.5)$

$= 0.5 \cap 0.9 \cap 0.6 \cap 0.5 = 0.5$

$D(a_4) = D(G_{ws}) = (0.1 \cup 0.8) \cap (0.2 \cup 0.4) \cap (0.5 \cup 0.9) \cap (0.2 \cup 0.4)$

$= 0.8 \cap 0.4 \cap 0.9 \cap 0.4 = 0.4$

$D(a_5) = D(T_{ws}) = (0.1 \cup 0.1) \cap (0.2 \cup 0.4) \cap (0.5 \cup 0.3) \cap (0.2 \cup 0.1)$

$= 0.1 \cap 0.4 \cap 0.7 \cap 0.2 = 0.7$

$D = \max \{D(a_1), D(a_2), D(a_3), D(a_4), D(a_5),\}$

$= \text{Max} \{0.5, 0.6, 0.5, 0.4, 0.5\} = 0.6$

Thus, I have chosen the second alternative a1 Kolar water supply as water supply scheme for ward number 54 under preference number 1.

Now in the second scenario the planner has given a different set of preferences by BMC as shown in the preference value Fig. 2.

$b_1=0.7$ $b_2=0.2$ $b_3=0.3$ $b_4=0.8$

$\bar{b}_1=0.3$ $\bar{b}_2=0.8$ $\bar{b}_3=0.7$ $\bar{b}_4=0.2$

$D (a_1) = D (U_{lws}) = (\bar{b}_1 \cup o_1) \cap (\bar{b}_2 \cup o_2) \cap (\bar{b}_3 \cup o_3) \cap (\bar{b}_4 \cup o_4)$

$= (0.3 \cup 0.9) \cap (0.8 \cup 0.5) \cap (0.3 \cup 0.9) \cap (0.2 \cup 0.8)$

$= 0.9 \cap 0.8 \cap 0.9 \cap 0.8 = 0.8$

$D (a_2) = D (K_{ws}) = (0.3 \cup 0.8) \cap (0.8 \cup 0.7) \cap (0.3 \cup 0.8) \cap (0.2 \cup 0.6)$

$= 0.8 \cap 0.8 \cap 0.8 \cap 0.6 = 0.6$

$D (a_3) = D (N_{ws}) = (0.3 \cup 0.5) \cap (0.8 \cup 0.9) \cap (0.7 \cup 0.6) \cap (0.2 \cup 0.5)$

$= 0.5 \cap 0.9 \cap 0.7 \cap -0.5 = 0.5$

$D (a_4) = D (G_{ws}) = (0.3 \cup 0.8) \cap (0.8 \cup 0.4) \cap (0.7 \cup 0.9) \cap (0.2 \cup 0.4)$

$= 0.8 \cap 0.8 \cap 0.9 \cap 0.4 = 0.4$

$D (a_5) = D (T_{ws}) = (0.3 \cup 0.1) \cap (0.8 \cup 0.4) \cap (0.7 \cup 0.3) \cap (0.2 \cup 0.1)$

$= 0.3 \cap 0.8 \cap 0.7 \cap 0.2 = 0.2$

Therefore, $D^* = \max \{D (a_1), D (a_2), D (a_3), D (a_4), D (a_5),\}$

$= \text{Max} \{0.8, 0.6, 0.5, 0.4, 0.2\} = 0.8$

From the above results upper Lake water supply is most suitable for water supply to ward number 54.

3. CONCLUSIONS

Water supply has been affected even in the low lying areas in old Bhopal. People are either hiring water tankers at exorbitant rates or are being forced to abandon their houses. The situation is also alarming in some districts and remote rural areas of Madhya Pradesh. To overcome this alarming situation Master Plan or Development Plans by Town and Country Planning Departments have already been formulated for every twenty years, in accordance with provisions of acts for various towns. The exercise commences with base maps, surveys and should include all determinants related to utility and services specially water supply. While urban economy continues to grow, towns and cities suffer from overcrowding, congestion, slums, lack of civic services without high quality physical environment. Hence, the need is to develop holistic urban information system comprising of spatial and attribute data in the form of determinants and indicators. In conventional master plan preparation, there is often mismatch of spatial units in terms of multiplicity. While urban areas contribute 60 percent of the GDP, yet tackling problems of water supply, housing, transportation, infrastructure, pollution, etc; remains unresolved. This has created a huge challenge for urban planners and managers of urban areas requiring systematic compilation of spatial and attributes data and related information. Environmental analysis supports planning and management requirements of various city government departments dealing with utilities and services. Multi-information based Master Plans for all selected urban settlements by an integrated analysis of land use, growth potential, population distribution, network and socio-economic parameters is the need of the hour, which should help in building models for providing services effectively. Preparation of under ground water potential maps, preparation of hazard potential maps for planning disaster mitigation and preparedness and incorporation of the existing master plans into new database is required.

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